

NJL model with infrared confinement

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Abstract. We consider an extended Nambu–Jona-Lasinio (NJL) model for the light meson sector of QCD where unphysical quark production thresholds are excluded by an infrared cut-off on the momentum integration within quark loop diagrams. This chiral quark model conserves the low energy theorems. The infrared cut-off is fixed selfconsistently by the dynamically generated quark mass (quark condensate). The masses and decay widths of the σ - and ρ -mesons are described in the model.

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1. Introduction

The Nambu–Jona-Lasinio (NJL) model is a convenient semiphenomenological quark model for the description of the low energy meson physics [1, 2, 3, 4, 5]. Within this model the mechanism of spontaneous breaking of chiral symmetry (SBCS) is realized in a simple and transparent way, and the low energy theorems are fulfilled.

Unfortunately, the ordinary NJL model fails to prevent hadrons from decaying into free quarks, which makes the realistic description of hadron properties on their mass shell questionable. The exact solution of this problem seems to be a very difficult task. However, different methods have been proposed for its solution [6, 7, 8, 9, 10]. In the present work we discuss a new approach which is close to that suggested in [10] where an infrared (IR) cut-off has been used for the construction of a quark propagator without poles. In our approach, the quark propagator is of the usual form (with quasiparticle pole) but due to the IR cut-off the pole does not lie within the integration interval for the quark loops. This method of taking into account the phenomenon of confinement is based on the idea of combining the NJL and bag models [11].

Thus, together with the ultraviolet (UV) cut-off, which is necessary for the elimination of the UV divergences, we introduce the IR cut-off and thereby divide the momentum space into three domains. In Fig. 1 these domains are represented in the coordinate space.

The first domain corresponds to short distances (large momenta), where quarks are not confined and the chiral symmetry is not spontaneously broken. This domain is excluded by the UV cut-off Λ .

The second domain corresponds to long distances (IR region) and here we have the confinement of quarks. We truncate this region from the integration over the internal momenta in quark loops, following thereby the idea of the bag model. For this purpose we introduce a new parameter λ .

Finally, there remains only the third domain ($\lambda^2 \leq \vec{p}^2 \leq \Lambda^2$) where SBCS takes place, the quark condensate exists and the quark loops have no imaginary parts. In other words, quark-antiquark thresholds do not appear when calculating quark loops even if the mass of the decaying meson exceeds the effective mass of the free quark-antiquark state. Therefore we can use quark propagators with constant, momentum independent, masses (constituent masses).

The first attempt to construct a NJL model of this type has been made in [12] where only the scalar and pseudoscalar mesons were considered. Now we suggest a more general version of this model where the scalar, pseudoscalar and vector mesons can be described and the possibility of $\pi - a_1$ -transitions is taken into account.

The paper is organized as follows. In Sec. 2 we give the effective chiral quark Lagrangian and the gap equation describing SBCS. The pion mass formula is also obtained and it is shown that the pion is a Goldstone boson in the chiral limit. In Sec. 3 the scalar meson (σ) is considered, and it is demonstrated that the quark loop with two σ -meson legs does not have the imaginary part if we use the IR cut-off. The

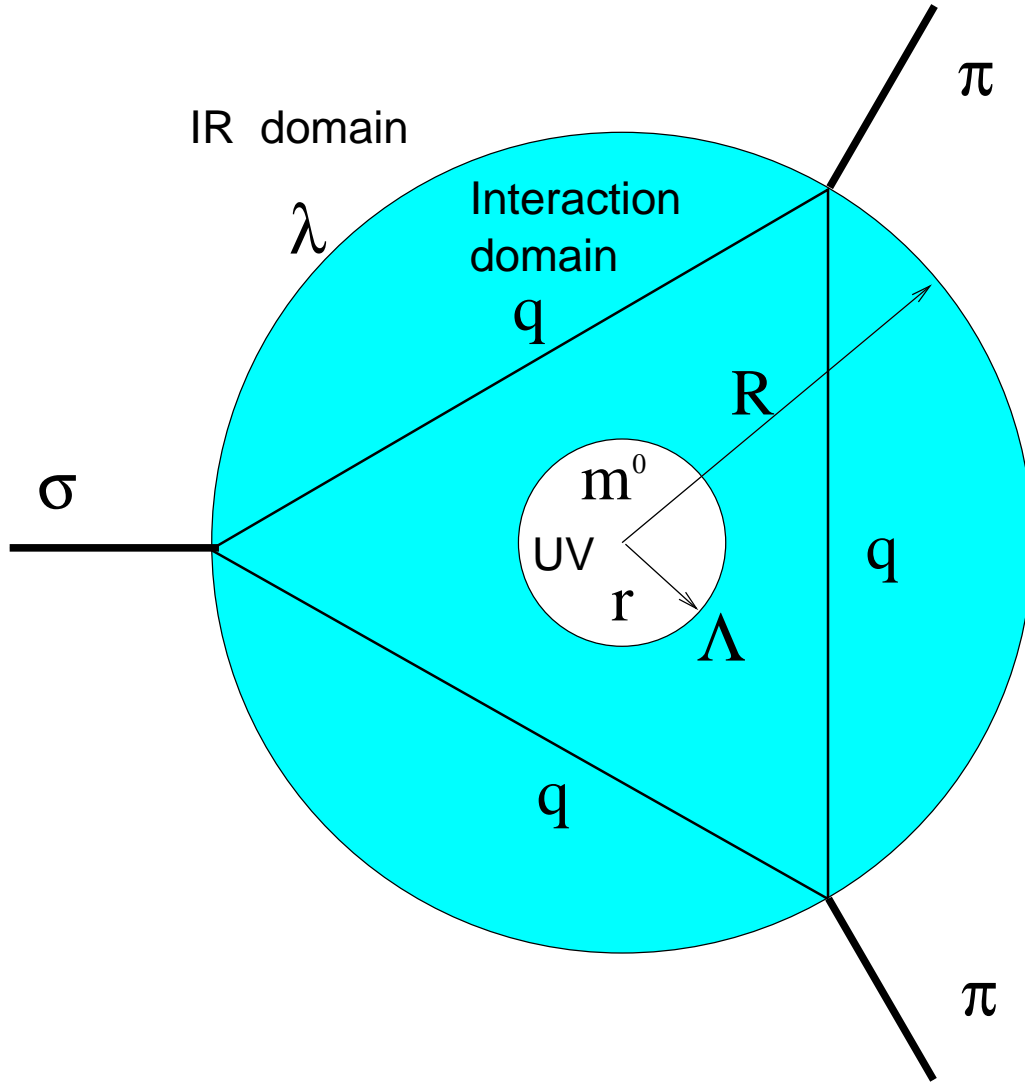


Figure 1. Three domains in the momentum space, defined by the UV and IR cut-offs.

model parameters are fitted in Sec. 4. There we consider the decay $\rho \rightarrow 2\pi$ through the triangle quark loop where the quark-antiquark threshold does not appear when the IR cut-off is applied. In Sec. 5 the σ -meson mass and the decay $\sigma \rightarrow 2\pi$ are estimated. In the last section we discuss the obtained results and give a short plan of applying this model for the investigation of the behavior of the mesons in a hot and dense medium in the vicinity of critical point. The values of the model parameters, the σ -meson mass and the $\sigma \rightarrow 2\pi$ decay width are given in Table I for different values of λ .

2. SU(2)×SU(2) Lagrangian, gap equation and pion mass formula

Let us consider an SU(2)×SU(2) NJL model defined by the Lagrangian

$$\begin{aligned} \mathcal{L}_q = & \bar{q}(i\partial - m^0)q + \frac{G_1}{2} [(\bar{q}q)^2 + (\bar{q}i\gamma_5\vec{\tau}q)^2] \\ & - \frac{G_2}{2} [(\bar{q}\gamma_\mu\vec{\tau}q)^2 + (\bar{q}\gamma_5\gamma_\mu\vec{\tau}q)^2] . \end{aligned} \quad (1)$$

After the bosonization of the four-fermion model (1) one obtains its equivalent representation in terms of the scalar (σ), pseudoscalar ($\vec{\pi}$), vector (ρ_μ) and axial-vector ($a_{1\mu}$) mesons

$$\begin{aligned} \mathcal{L}_{meson} = & -\frac{\tilde{\sigma}^2 + \vec{\pi}^2}{2G_1} + \frac{\vec{\rho}_\mu^2 + \vec{a}_{1\mu}^2}{2G_2} \\ & - i\text{Tr} \ln \left\{ 1 + \frac{1}{i\partial - m} [\sigma + i\gamma_5\vec{\tau}\vec{\pi} + \vec{\tau}\hat{\rho}_\mu + \gamma_5\vec{\tau}\hat{a}_{1\mu}] \right\} . \end{aligned} \quad (2)$$

Here, the scalar fields σ and $\tilde{\sigma}$ are connected by the relation

$$-m^0 + \tilde{\sigma} = -m + \sigma , \quad (3)$$

where m^0 is the current quark mass, m is the constituent quark mass; and the vacuum expectation of σ vanishes: $\langle\sigma\rangle_0 = 0$. Then, from the condition

$$\left. \frac{\delta\mathcal{L}}{\delta\sigma} \right|_{\sigma=0, \vec{\pi}=0} = 0 , \quad (4)$$

one obtains the gap equation||

$$\begin{aligned} m^0 &= m(1 - 8G_1 I_1^{(\Lambda\Lambda)}(m)) \\ &\approx m(1 - 8G_1 I_1^{(0\Lambda)}(m)) \\ &= m + 2G_1 \langle\bar{q}q\rangle_0 \end{aligned} \quad (5)$$

where $\langle\bar{q}q\rangle_0$ is the quark condensate. $I_1^{(ab)}(m)$ is obtained from the Λ^2 -divergent integral

$$I_1(m) = -i \frac{N_c}{(2\pi)^4} \int \frac{d^4k}{m^2 - k^2 - i\varepsilon} \quad (6)$$

by applying the 3-dimensional UV ($b = \Lambda$) and IR ($a = \lambda$) cut-offs

$$\begin{aligned} I_1^{(\Lambda\Lambda)}(m) &= \frac{N_c}{(2\pi)^2} \int_\lambda^\Lambda dk \frac{k^2}{E(k)} \\ &= \frac{N_c m^2}{8\pi^2} \left[x\sqrt{x^2 + 1} - \ln(x + \sqrt{x^2 + 1}) \right] \Big|_{\lambda/m}^{\Lambda/m} , \end{aligned} \quad (7)$$

where $E(k) = \sqrt{k^2 + m^2}$ and N_c is the number of colors.

|| Here, the dependence of the integral $I_1(m)$ on λ can be neglected since: 1) the value of the integral is defined by the UV cut-off Λ and 2) $I_1(m)$ does not depend on external momenta and, therefore, has not the imaginary part. Hence, there is no need for an IR cut-off.

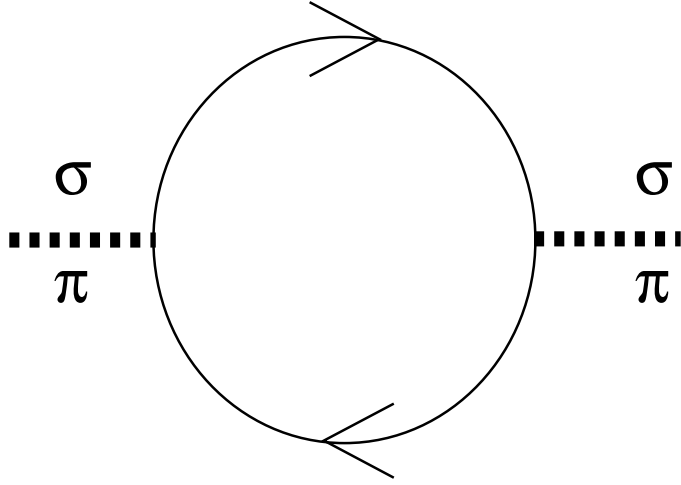


Figure 2. The quark-loop diagram for the polarization operator of σ and π .

Now let us consider the free part of the Lagrangian (2) for pion fields in the quark one-loop approximation (see Fig. 2)¶

$$\mathcal{L}_\pi^{(2)} = -\frac{\vec{\pi}^2}{2} \left\{ \frac{1}{G_1} - 8I_1^{(\lambda\Lambda)}(m) - 4p^2 I_2^{(\lambda\Lambda)}(p^2, m) \right\}, \quad (8)$$

where $I_2^{(\lambda\Lambda)}(p^2, m)$ is a logarithmically divergent integral

$$\begin{aligned} I_2^{(\lambda\Lambda)}(p^2, m) &= -i \frac{N_c}{(2\pi)^4} \int \frac{d^4k}{(m^2 - k^2 - i\varepsilon)(m^2 - (k-p)^2 - i\varepsilon)} \\ &= \frac{N_c}{2\pi^2} \int_\lambda^\Lambda dk \frac{k^2}{E(4E^2 - p^2 + i\varepsilon)}. \end{aligned} \quad (9)$$

In order to express (8) through physical fields, we renormalize the pions

$$\vec{\pi} = g_\pi(M_\pi) \vec{\pi}^r, \quad g_\pi(M_\pi) \approx g_\pi(0) = [4I_2^{(\lambda\Lambda)}(0, m)]^{-1/2}. \quad (10)$$

Here, as the pion mass M_π is small, we can approximate the loop integral by $I_2^{(\lambda\Lambda)}(p^2 = M_\pi^2, m) \approx I_2^{(\lambda\Lambda)}(0, m)$, where

$$I_2^{(\lambda\Lambda)}(0, m) = \frac{N_c}{8\pi^2} \left[\ln(x + \sqrt{x^2 + 1}) - (1 + 1/x^2)^{-1/2} \right] \Big|_{\lambda/m}^{\Lambda/m}. \quad (11)$$

Moreover, for pions, an additional renormalization factor \sqrt{Z} appears when we take into account the $\pi - a_1$ -transitions [3]:

$$\bar{g}_\pi = g_\pi \sqrt{Z}, \quad Z^{-1} = 1 - \frac{6m^2}{M_{a_1}^2} \quad (12)$$

¶ The expression in the brackets can be written in the form $1/G_1 + \Pi_\pi(p)$, where $\Pi_\pi(p)$ is the polarization operator of the pion.

where $M_{a_1} = 1230$ MeV is the mass of the a_1 -meson. Thus, we obtain the following expression for the pion mass:

$$M_\pi^2 = \bar{g}_\pi^2 \left[\frac{1}{G_1} - 8I_1^{(\lambda\Lambda)}(m) \right], \quad (13)$$

which can be given the form of the Gell-Mann–Oakes–Renner relation

$$M_\pi^2 \approx -2 \frac{m^0 \langle \bar{q}q \rangle_0}{F_\pi^2}, \quad (14)$$

where the Goldberger-Treiman relation (21) and the gap equation (5) have been used. We can see that this pion mass formula is in accordance with the Goldstone theorem since for $m^0 = 0$ the pion mass vanishes and it becomes a Goldstone boson.

3. The σ -meson and IR confinement

The free part of the Lagrangian (2) for the σ -meson in the one-loop approximation (see Fig. 2) has the following form

$$\mathcal{L}_\sigma^{(2)} = -\frac{\sigma^2}{2} \left\{ \frac{1}{G_1} - 8I_1^{(\lambda\Lambda)}(m) - 4(p^2 - 4m^2)I_2(p^2, m) \right\}. \quad (15)$$

After the renormalization of the σ field

$$\sigma = g_\sigma(M_\sigma)\sigma^r, \quad g_\sigma(M_\sigma) = [4I_2^{(\lambda\Lambda)}(M_\sigma, m)]^{-1/2} \quad (16)$$

we obtain the expression for the σ -meson mass

$$\begin{aligned} M_\sigma^2 &= g_\sigma^2(M_\sigma) \left[\frac{1}{G_1} - 8I_1^{(\lambda\Lambda)}(m) \right] + 4m^2 \\ &= r^2 M_\pi^2 + 4m^2, \quad \left(r = \frac{g_\sigma(M_\sigma)}{g_\pi(M_\pi)} \right). \end{aligned} \quad (17)$$

Now let us consider more carefully the integral $I_2^{(\lambda\Lambda)}(M_\sigma, m)$:

$$I_2^{(\lambda\Lambda)}(M_\sigma^2, m) = \frac{N_c}{2\pi^2} \int_\lambda^\Lambda dk \frac{k^2}{E(4E^2 - M_\sigma^2 + i\varepsilon)} \quad (18)$$

When $\lambda = 0$ this integral has an imaginary part. Indeed, the integrand in (18) is singular when its denominator is equal to zero:

$$4E^2 - M_\sigma^2 = 4k^2 - r^2 M_\pi^2 = 0. \quad (19)$$

The imaginary part appears if the singularity ($k = \frac{r}{2} M_\pi$) lies within the integration interval. Therefore, if we apply the IR cut-off

$$\lambda = c m \quad \text{where} \quad c > \frac{r M_\pi}{2m}, \quad (20)$$

then $\Lambda > k > \lambda > \frac{r}{2} M_\pi$ and the integral is real, which means the absence of the quark-antiquark threshold, or confinement.

4. Model parameters

In this model we have five parameters: the constituent quark mass m , the scalar (pseudoscalar) four-quark coupling constant G_1 , the vector (axial-vector) four-quark coupling constant G_2 , the 3-momentum UV cut-off parameter Λ and the 3-momentum IR cut-off parameter λ .

Since we will use for the determination of these parameters only the four pion and ρ meson observables [13]: $M_\pi = 140$ MeV, $F_\pi = 92.4$ MeV, $M_\rho = 770$ MeV and $g_\rho^{\text{exp}} = 6.14$, the IR cut-off λ is an arbitrary parameter of our model which is to satisfy the condition (20). We have checked the sensitivity of the model to a variation of λ and the results are summarized in Table I. We will consider $\lambda = m$ as optimal choice since it does not only give acceptable σ meson properties (see next Section).⁺

In order to fix the output parameters we use the following four equations:

- 1) The Goldberger–Treiman relation

$$\frac{m}{F_\pi} = \bar{g}_\pi(0) = g_\pi(0)\sqrt{Z}, \quad (21)$$

- 2) The decay width of the process $\rho \rightarrow 2\pi$. The amplitude of this process is of the form

$$T_{\rho \rightarrow 2\pi} = i \frac{g_\rho^{\text{exp}}}{2} (p_{\pi^+} - p_{\pi^-})^\nu \rho_\nu^0 \pi^+ \pi^-. \quad (22)$$

In the one-loop approximation (see Fig. 3), we obtain the following expression for g_ρ^{exp}

$$g_\rho^{\text{exp}} = Z^{-1} g_\rho(M_\rho) \bar{g}_\pi^2(M_\pi) \left[4I_2^{(\lambda \Lambda)}(0, m) + \Delta \right] \quad (23)$$

$$= \left(\frac{2}{3} I_2^{(\lambda \Lambda)}(M_\rho^2, m) \right)^{-1/2} \left[1 + \frac{\Delta}{4I_2^{(\lambda \Lambda)}(0, m)} \right], \quad (24)$$

where $g_\rho(M_\rho) = [\frac{2}{3} I_2^{(\lambda \Lambda)}(M_\rho^2, m)]^{-1/2}$ (see [3]) and $\Delta = \frac{3}{8\pi^2} \left(1 + \frac{2M_\pi^2}{3m^2} \right)$ is the finite part of the quark triangle diagram (Fig. 3). The factor Z^{-1} appears due to the $\pi - a_1$ -transitions (see [3]). From these two equations one can find m and Λ .

- 3) The coupling constant G_1 is defined by the mass formula

$$M_\pi^2 = \bar{g}_\pi^2 \left[\frac{1}{G_1} - 8I_1^{(\lambda \Lambda)}(m) \right]. \quad (25)$$

- 4) The coupling constant G_2 is found from the mass formula for M_ρ [3]

$$M_\rho^2 = \frac{g_\rho^2(M_\rho)}{G_2} = \frac{3}{8G_2 I_2^{(\lambda \Lambda)}(M_\rho^2, m)}. \quad (26)$$

By means of the gap equation (5) we define the current quark mass m^0 . The result of the above described parameter fixing procedure is summarized in Table I for different choices of the IR cut-off λ given by the ratio λ/m .

⁺ This is also suggested from a comparison with the simple confining model by Munczek and Nemirovsky [14], where in the chiral limit the lower limit of quark four-momenta p for which quarks are deconfined is given by the dynamical quark mass function at $p^2 = 0$.

Table 1. Model parameters, the mass of the σ -meson and its decay width for different values of the ratio of the IR cut-off to the constituent quark mass λ/m .

λ/m	m [MeV]	m^0 [MeV]	Λ [GeV]	G_1 [GeV] ⁻²	G_2 [GeV] ⁻²	$-\langle\bar{q}q\rangle^{1/3}$ [MeV]	M_σ [MeV]	Γ_σ [MeV]
0.7	330	1.54	1.11	3.108	9.683	298	670	1280
0.8	310	1.58	1.13	2.967	8.629	296	630	970
0.9	300	1.49	1.18	2.698	8.949	302	608	830
1.0	290	1.45	1.21	2.555	9.222	304	590	721
1.1	280	1.37	1.26	2.339	9.322	309	570	614
1.2	270	1.32	1.33	2.079	9.273	318	550	536

5. The σ -meson mass and the decay $\sigma \rightarrow 2\pi$

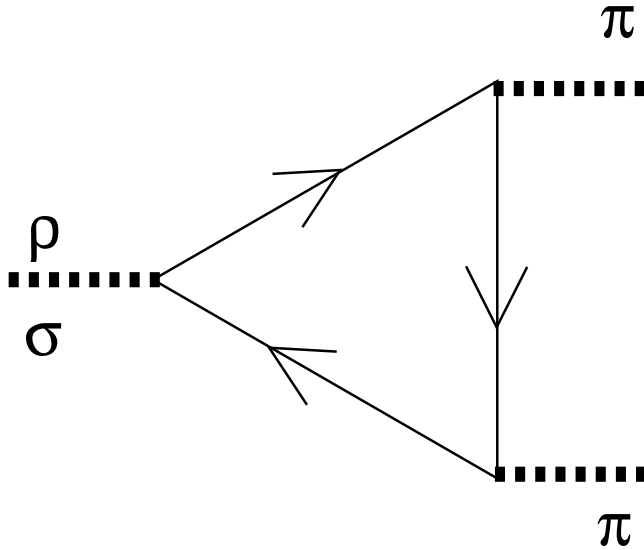
The mass of the σ -meson is given by eq. (17). Using this formula for the IR cut-off $\lambda = m$ we obtain

$$M_\sigma = 590 \text{ MeV}. \quad (27)$$

The decay $\sigma \rightarrow 2\pi$ occurs through the quark triangle diagram (see Fig. 3).

This diagram also satisfies the confinement condition (20) if we use the IR cut-off $\lambda = m$. The amplitude of the process $\sigma \rightarrow 2\pi$ has the form

$$T_{\sigma \rightarrow 2\pi} = 8mg_\sigma(M_\sigma)\bar{g}_\pi^2(M_\pi)[I_2^{(\lambda\Lambda)}(M_\sigma^2, m) + \mathcal{J}(M_\sigma, M_\pi, m)]\sigma\vec{\pi}^2, \quad (28)$$

**Figure 3.** The triangle quark diagram describing the decay of the ρ - and σ -mesons into two pions.

where

$$\mathcal{J}(M_\sigma, M_\pi, m) = \frac{1}{2}(M_\sigma^2 - 2M_\pi^2)I_3(p_1, p_2, m) \Big|_{p_1^2=p_2^2=M_\pi^2} \quad (29)$$

and

$$I_3(p_1, p_2, m) = \frac{-iN_c}{(2\pi)^4} \int \frac{d^4k}{(k^2 - m^2)((k + p_1)^2 - m^2)((k - p_2)^2 - m^2)}. \quad (30)$$

Neglecting the external pion momenta in $I_3(p_1, p_2, m)$ we obtain

$$\begin{aligned} I_3(p_1, p_2, m) &\approx \frac{-iN_c}{(2\pi)^4} \int \frac{d^4k}{(k^2 - m^2)^3} = -\frac{3N_c}{32\pi^2} \int_\lambda^\Lambda dk \frac{k^2}{E^5} \\ &= -\frac{N_c}{32\pi^2 m^2} \left[\left(1 + \frac{m^2}{\Lambda^2}\right)^{-3/2} - \left(1 + \frac{m^2}{\lambda^2}\right)^{-3/2} \right]. \end{aligned} \quad (31)$$

Then the decay width of the σ -meson is equal to

$$\Gamma_{\sigma \rightarrow 2\pi} = \frac{3}{2\pi} \left(\frac{m^3}{g_\sigma(M_\sigma)F_\pi^2} \right)^2 \frac{(1 + \delta)^2}{M_\sigma^2} \sqrt{M_\sigma^2 - 4M_\pi^2} \approx 720 \text{ MeV}, \quad (32)$$

where

$$\delta = \frac{\mathcal{J}(M_\sigma, M_\pi, m)}{I_2^{(\lambda, \Lambda)}(0, m)} \approx -0.22. \quad (33)$$

Therefore, one can see that our estimates for the σ -meson mass and its decay width are in agreement with the experimental data [13] (see also [15, 16]):

$$M_\sigma^{\text{exp}} = (400 - 1200) \text{ MeV}, \quad \Gamma_\sigma^{\text{exp}} = (600 - 1000) \text{ MeV}. \quad (34)$$

From this one can conclude that the NJL model with the IR cut-off satisfies both of the low energy theorems together with SBCS and describes the low-energy physics of the scalar, pseudoscalar and vector mesons.

6. Discussion and conclusion

In this paper we have investigated the extension of the NJL model for the light nonstrange meson sector of QCD, where the interaction of u - and d -quarks is represented by four-fermion vertices and the phenomenon of quark confinement is taken into account. This extension of the NJL model describes the properties of the π , ρ and σ -mesons in good agreement with the experiment and with the low-energy theorems. The model parameters are obtained by fitting the model so that it reproduces the experimental values of the pion and ρ -meson masses, the pion decay constant F_π and the ρ -meson decay constant g_ρ . Moreover, it was shown that for the π -, ρ - and σ -mesons the non-physical quark-antiquark thresholds do not appear if the IR cut-off is applied.

The prediction of the σ -meson mass and its decay width for different values of IR cut-off is given in Table I together with the model parameters. From this table and the experimental data [13] one can conclude that the parameter λ is allowed to have values within the interval $0.8 \leq \lambda/m \leq 1.2$. The value of the current quark mass turned out to

be too low because in our model the quark condensate is greater than its conventional value $-(250\text{MeV})^3$. This can be seen from eq. (14).

Insofar as the NJL model is a semiphenomenological model based on the effective chiral four-quark interaction motivated by QCD on the phenomenological level, the introduction of quark confinement in our model by means of an IR cut-off (without considering the gluon exchanges, instanton interactions etc.) is in the spirit of this model.

An interesting application of our model is the description of meson properties in a hot and dense medium. The standard NJL model has been already used for this purpose [5, 17, 18], where the temperature dependence of the masses of quarks and mesons and of the Yukawa coupling constants was found. The IR cut-off λ is expressed through the constituent quark mass m (or the quark condensate $\langle\bar{q}q\rangle_0$), which decreases when the temperature (T) and chemical potential (μ) increase. Therefore, the IR cut-off will also decrease with T and μ . This will at length result in the deconfinement of quarks near the critical point. The temperature at which deconfinement takes place can be found from the condition (see also eq. (19))

$$4E_{\text{low}}^2 - M_{\text{meson}}^2 = 4(1 + c^2)m^2 - M_{\text{meson}}^2 \leq 0 \quad (35)$$

where E_{low} is the lowest energy of the quark in the quark loop. Then, for the pion, the temperature of deconfinement follows from the condition (35)

$$m(T_{\pi}^{\text{dec}}) \approx \frac{M_{\pi}}{2\sqrt{1 + c^2}}. \quad (36)$$

In the vicinity of the critical point, the constituent quark mass decreases with T very quickly, and the pion mass slowly increases (see [18]). Therefore, when the constituent quark mass is as light as 40–50 MeV, the decay of pion into free quarks becomes possible.

For the σ -meson we obtain a lower value of T^{dec} :

$$m(T_{\sigma}^{\text{dec}}) \approx \frac{M_{\pi}}{c}r. \quad (37)$$

The decay channel $\sigma \rightarrow 2\pi$ is closed when $M_{\sigma} \leq 2M_{\pi}$ (see eq. (32)) and then

$$m(T_{\sigma \rightarrow 2\pi}) \approx \frac{\sqrt{3}}{2}M_{\pi}r \sim 100 \text{ MeV}. \quad (38)$$

Note that, first of all, the quark deconfinement occurs for the ρ -meson, where we have the lowest T^{dec} (see Eqs. (35) and [18])

$$m(T_{\rho}^{\text{dec}}) \approx \frac{M_{\rho}}{2\sqrt{1 + c^2}} \approx 250 \text{ MeV}. \quad (39)$$

Thus, we have the following picture. At low T and μ the σ -meson is unstable since it has a large decay width (32) into two pions. The pion is stable since electroweak decay channels can be neglected here in comparison to the strong ones. The ρ -meson takes an intermediate position between σ and π , having the $\rho \rightarrow 2\pi$ strong decay width 150 MeV. With T increasing, there opens an additional decay channel: $\rho \rightarrow \bar{q}q$. At higher T , there exists an interval in the temperature scale where the decays of σ both into 2π and into $\bar{q}q$ are forbidden, and the σ -meson turns out to be a stable particle.

Next, when $T > T_{\sigma}^{\text{dec}}$ the σ -meson is allowed to decay into a $\bar{q}q$ pair and only the pion remains to be stable. Finally, near the critical point, when $T \geq T_{\pi}^{\text{dec}}$, all the particles decay into free quarks.

The whole process of deconfinement is reflected in the following sequence of inequalities

$$T_{\rho}^{\text{dec}} < T_{\sigma \rightarrow 2\pi} < T_{\sigma}^{\text{dec}} < T_{\pi}^{\text{dec}} \quad (40)$$

where we have four different temperatures separating different phases. Therefore, one can see that the transition of the hadron matter to the quark-gluon plasma occurs not abruptly but in a smooth manner.

In our further work we are going to make a more careful investigation of these processes, which can play an important rôle for the explanation and prediction of signals coming from ultrarelativistic heavy-ion collisions and witnessing the chiral symmetry restoration and the quark deconfinement in hadron matter at the transition to the quark-gluon plasma. Of particular interest is the study of quark substructure effects on ρ meson properties which is possible within the present model since their modifications at the suspected QCD phase transition are crucial for understanding, e.g., the low-mass dilepton enhancement as observed by the CERES collaboration [19].

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